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Transient Characteristics of Induction Motor Due to Reconnection of Variable Frequency Supply

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Synopsis

This work is concerned with induction motor transients resulting from disconnecting the motor from one source and reconnecting it to another. The torque and current transients depend primarily upon the residual motor voltage, the phase of the reapplied voltage at the instant of reconnection, the ratio of the frequency of the two sources, and the load inertia. Good agreement is found between theoretical results based upon standard transient analysis and experimental evaluation of the torque, current and speed transients. In this study, all measurements and tests are performed on a 200V, 1.5kW three-phase induction motor, and investigated under the condition that the ratio of the reapplied voltage to the frequency is constant.

1. Introduction

The availability of variable frequency and variable voltage static power converters has made it possible to develop a variety of variable speed a.c. motor drives. The use of variable speed drives to control induction motors has received particular attention because induction motors are relatively cheap and require little maintenance.

Polyphase induction motors can usually be operated at any one of a set of steady state speeds by utilizing various combinations of pole connections. In many applications, it suffices to restrict the use of the variable frequency drive to controlling the motor during a transition from one of these speed levels to the other. In such cases, the motor is a) disconnected from the line, b) connected to the converter, c) driven to the next desired speed level, d) disconnected from the converter and, finally, e) is reconnected to the line with the pole connections appropriate for the new speed level at line frequency. We shall refer subsequently to this sequence of events as a disconnect-reconnect operation.

Several authors[1,2,3] have studied the steady state properties of induction motors driven from variable frequency supplies, and Berg[4] has studied the problem of minimizing the transition time. References 5–8 are examples of studies of the transients resulting from the disconnect-reconnect operation. Reference 9 deals with transients resulting from star/delta transformations. The object of this study is to evaluate the motor transients experimentally and compare these with the results obtained from analytical and simulation studies. In this study, the authors investigate under the condition that the ratio of reconnected supply voltage to the frequency is approximately 4.0.

2. Analysis

2.1 Fundamental assumptions and conditions.

Subsequent transient analysis is based upon the following fundamental assumptions about the induction motor.

(1) The distributed windings produce a sinusoidal distribution of m. m. f.

(2) Eddy currents and hysteresis effects are negligible and the airgap is uniform.

(3) The flux paths do not saturate.

(4) The winding resistances and inductances are constant independent of frequency and speed.

Conditions on the reapplied voltage are as follows:

(1) The reapplied voltages are balanced and do not fluctuate due to reconnection.

(2) Relationship between the reapplied voltage V and the frequency is held constant, $V/f \simeq 4[V/Hz]$.

2.2 Voltage and current equations.

Figure 1 shows the instantaneous-positive-sequence equivalent circuit of a three phase induction motor[9][10]. The positive-sequence voltage equations for the stator and the rotor are accordingly.

$$v_{s1}=(R_s+pL_s)i_{s1}+pL_m i_{r1} \tag{1}$$

$$0=(R_r+pL_r)i_{r1}+pL_m i_{s1}-e_{v1} \tag{2}$$

$$e_{v1}=jw_m(L_r i_{r1}+L_m i_{r1}) \tag{3}$$

where the time t is measured in reference to the instant of reconnection.

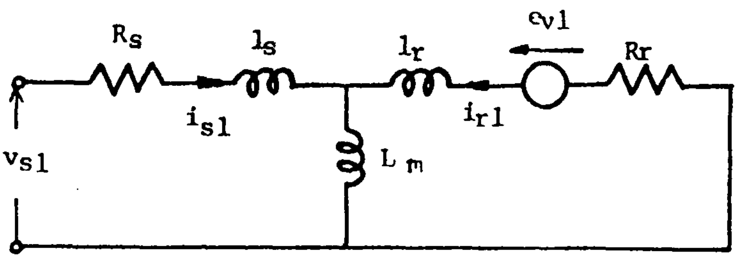

Fig. 1 Instantaneous positive-sequence equivalent circuit

Taking the Laplace Transform on both sides of Eq. (1)～(3) and solving $I_{s1}(s)$ and $I_{r1}(s)$, we have

$$I_{s1}(s)=[\{V_{s1}(s)+L_m i_{r10}+L_s i_{s10}\}\{R_r+(s-j\omega_m)L_r\}-(L_r i_{r10}+L_m i_{s10})sL_m]/\Delta(s) \qquad (4)$$

$$I_{r1}(s)=[(L_m i_{s10}+L_r i_{r10})(R_s+sL_s)-(V_{s1}(s)+L_s i_{s10}+L_m i_{r10})(s-j\omega_m)L_m]/\Delta(s) \qquad (5)$$

$$\Delta(s)=(L_sL_r-L^2{}_m)(s+\alpha_1-j\beta_1)(s+\alpha_2-j\beta_2)\ \neq 0 \qquad (6)$$

where

$$\alpha_1,\ \alpha_2=\frac{1}{2}\Big[\frac{L_sR_r+L_rR_s}{L_sL_r-L^2_m}\mp\sqrt{\frac{1}{2}(\sqrt{A^2+B^2}+A)}\Big] \qquad (7)$$

$$\beta_1,\ \beta_2=\frac{1}{2}\Big[\omega_m\mp\sqrt{\frac{1}{2}(\sqrt{A^2+B^2}-A)}\Big] \qquad (8)$$

$$A=\Big(\frac{L_sR_r+L_rR_s}{L_sL_r-L^2_m}\Big)^2-\frac{4R_sR_r}{L_sL_r-L^2_m}-\omega^2{}_m \qquad (9)$$

$$B=2\omega_m(L_sR_r-L_rR_s)/(L_sL_r-L^2{}_m) \qquad (10)$$

The line voltages are balanced, and the Laplace transform of the reapplied positive-sequence voltage in Eq. (4) and (5) is consequently

$$V_{s1}=\sqrt{\frac{3}{2}}\ \frac{V_1\varepsilon^{j\xi}}{s-j\omega} \qquad (11)$$

Since the stator coils are open prior to reconnection, the initial condition of i_{s1} is

$$i_{s10}=0 \qquad (12)$$

and the residual voltage of the reference stator phase is

$$e_a=\sqrt{2}\ E(1-S)\varepsilon^{-\frac{R_r}{L_r}t'}\ cos\{(1-S)\omega' t'+\phi'\} \qquad (13)$$

where $\omega_m=(1-S)\omega'$.

Since the residual voltage in the stator is produced by the rotor current, we have next equation for reference phase stator voltage and rotor current

$$L_m\frac{di_{ra}}{dt'}=e_a \qquad (14)$$

The duration of disconnection is relatively short and the time constant L_r/R_r is also negligibly small compared to the mechanical time constant J/D of the rotor. As a result, the solution of Eq. 14 is

$$i_{ra}=\frac{\sqrt{2}(1-S)E}{\omega_m L_m}\varepsilon^{-\frac{R_r}{L_r}t'}\ sin(\omega_m t'+\phi') \qquad (15)$$

The positve-sequence current i_{r1} is obtained by using Eq. (15).

$$i_{r1}=\sqrt{\frac{3}{2}}\ \frac{(1-S)E}{\omega_m L_m}\varepsilon^{-\frac{R_r}{L_r}t'}\ \varepsilon^{j(\omega_m t'+\phi'-\frac{\pi}{2})} \qquad (16)$$

Substituting the time interval t_1' of disconnection, slip S_0 and angular velocity ω_{m_0} of rotor at the instant of reconnection into Eq. (16), we obtain

$$E_0=(1-S_0)E\ \varepsilon^{-\frac{R_r}{L_r}t_1'} \qquad (17)$$

The initial value of i_{r1} is obtained from Eq. (16),

$$i_{r10}=\sqrt{\frac{3}{2}}\ \frac{E_0}{\omega_{m_0}L_m}\varepsilon^{j(\phi-\frac{\pi}{2})} \qquad (18)$$

where ϕ is the initial phase angle of the residual voltage referred to the reconnected source voltage.

Using Eq. (6) through Eq. (12) and Eq. (18), and taking the inverse Laplace transform of Eq. (4) and (5), we get the complete solutions for instantaneous-positive-sequence stator and rotor currents

$$i_{s1}=\sqrt{\frac{3}{2}}\delta[V_1\varepsilon^{j\xi}(\frac{R_r/L_r+j(\omega-\omega_m)}{\{\alpha_1+j(\omega-\beta_1)\}\{\alpha_2+j(\omega-\beta_2)\}}\varepsilon^{j\omega t}$$
$$+\frac{R_r/L_r-\alpha_1+j(\beta_1-\omega_m)}{\{-\alpha_1+j(\beta_1-\omega)\}\{\alpha_2-\alpha_1)+j(\beta_1-\beta_2)}\varepsilon^{(-\alpha_1+j\beta_1)t}$$
$$+\frac{R_r/L_r-\alpha_2+j(\beta_2-\omega_m)}{\{-\alpha_2+j(\beta_2-\omega)\}\{\alpha_1-\alpha_2)+j(\beta_2-\beta_1)\}}\varepsilon^{(-\alpha_2+j\beta_2)t})+\frac{E_0}{\omega_{m0}}\varepsilon^{j(\phi-\frac{\pi}{2})}$$
$$\times(\frac{R_r/L_r-j\omega_m}{\alpha_2-\alpha_1+j(\beta_1-\beta_2)}\varepsilon^{(-\alpha_1+j\beta_1)t}+\frac{R_r/L_r-j\omega_m}{\alpha_1-\alpha_2+j(\beta_2-\beta_1)}\varepsilon^{(-\alpha_2+j\beta_2)t})] \tag{19}$$

$$i_{r1}=\sqrt{\frac{3}{2}}\ \delta[\frac{L_m}{L_r}V_1\varepsilon^{j\xi}(\frac{j(\omega_m-\omega)}{\{\alpha_1+j(\omega-\beta_1)\}\{\alpha_2+j(\omega-\beta_2)\}}\varepsilon^{j\omega t}$$
$$+\frac{\alpha_1-j(\beta_1-\omega_m)}{\{(\alpha_2-\alpha_1)+j(\beta_1-\beta_2)\}\{-\alpha_1+j(\beta_1-\omega)\}}\varepsilon^{(-\alpha_1+j\beta_1)t}$$
$$+\frac{\alpha_2-j(\beta_2-\omega_m)}{\{(\alpha_1-\alpha_2)+j(\beta_2-\beta_1)\}\{-\alpha_2+j(\beta_2-\omega)\}}\varepsilon^{(-\alpha_2+j\beta_2)t})+\frac{E_0}{\omega_{m0}L_m}\varepsilon^{j(\phi-\frac{\pi}{2})}$$
$$\times\{\frac{R_s+L_s(-\alpha_1+j\beta_1)}{\alpha_2-\alpha_1+j(\beta_1-\beta_2)}\varepsilon^{(-\alpha_1+j\beta_1)t}+\frac{R_s+L_s(-\alpha_2+j\beta_2)}{\alpha_1-\alpha_2+j(\beta_2-\beta_1)}\varepsilon^{(-\alpha_2+j\beta_2)t}$$
$$-\frac{L^2_m}{L_r}(\frac{-\alpha_1+j(\beta_1-\omega_m)}{\alpha_2-\alpha_1+j(\beta_1-\beta_2)}\varepsilon^{(-\alpha_1+j\beta_1)t}+\frac{-\alpha_2+j(\beta_2-\omega_m)}{\alpha_1-\alpha_2+j(\beta_2-\beta_1)}\varepsilon^{(-\alpha_2+j\beta_2)t})\}] \tag{20}$$

where $\delta=L_r/(L_sL_r-L^2_m)$.

The stator and rotor current transients of the reference phase are calculated from Eq. (19) and (20) with the aid of transform equation (21).

$$i_s=\frac{2}{\sqrt{3}}R_e(i_{s1})$$
$$i_r=\frac{2}{\sqrt{3}}R_e(i_{r1}) \tag{21}$$

where the notation $\mathrm{Re}(i_{s1})$ implies the real part of i_{s1}. Equation 21 expand into

$$i_s(t)=\sqrt{2}\ \ \delta(V_1\{\eta_1cos(\omega t+\xi)-\eta_2sin(\omega t+\xi)\}$$
$$+[V_1\{\eta_3cos(\beta_1t+\xi)-\eta_4sin(\beta_1t+\xi)\}$$
$$+\frac{E_0}{\omega_{m0}}\{\eta_7cos(\beta_1t+\phi-\frac{\pi}{2})-\eta_8sin(\beta_1t+\phi-\frac{\pi}{2})\}]\varepsilon^{-\alpha_1t}$$
$$+[V_1\{\eta_5cos(\beta_2t+\xi)-\eta_6sin(\beta_2t+\xi)\}$$
$$-\frac{E_0}{\omega_{m0}}\{\eta_7cos(\beta_2t+\phi-\frac{\pi}{2})-\eta_8sin(\beta_2t+\phi-\frac{\pi}{2})\}]\varepsilon^{-\alpha_2t}) \tag{22}$$

and

$$i_r(t)=\sqrt{2}\ \ \delta(\frac{L_m}{L_r}V_1\{\eta_{10}sin(\omega t+\xi)-\eta_9cos(\omega t+\xi)\}$$

$$+[\frac{E_0}{\omega_{m0}L_m}\{\eta_{15}cos(\beta_1 t+\phi-\frac{\pi}{2})-\eta_{16}sin(\beta_1 t-\phi-\frac{\pi}{2})\}$$

$$-\frac{L_m}{L_r}\{\eta_{11}cos(\beta_1 t+\xi)-\eta_{12}sin(\beta_1 t+\xi)\}$$

$$-\frac{L_m E_0}{\omega_{m0}L_r}\{\eta_{19}cos(\beta_1 t+\phi-\frac{\pi}{2})-\eta_{20}sin(\beta_1 t+\phi-\frac{\pi}{2})\}]\varepsilon^{-\alpha_1 t}$$

$$+[\frac{E_0}{\omega_{m0}L_m}\{\eta_{17}cos(\beta_2 t+\phi-\frac{\pi}{2})-\eta_{18}sin(\beta_2 t+\phi-\frac{\pi}{2})\}$$

$$-\frac{L_m}{L_r}V_1\{\eta_{13}cos(\beta_2 t+\xi)-\eta_{14}sin(\beta_2 t+\xi)\}$$

$$-\frac{L_m E_0}{\omega_{m0}L_r}\{\eta_{21}cos(\beta_2 t+\phi-\frac{\pi}{2})-\eta_{22}sin(\beta_2 t+\phi-\frac{\pi}{2})\}]\varepsilon^{-\alpha_2 t}) \tag{23}$$

The coefficients, $\eta_i(i=1, 2, \cdots, 22)$ are functions of the equivalent circuit parameters of the induction motor, the electric angular velocity ω_m, and the angular frequency of the reconnected voltage ω.

2.3 Transient torque

The instantaneous electromagnetic torque on the rotor is

$$T=-2qI_m(L_m\bar{i}_{s1}i_{r1}) \tag{24}$$

where I_m means "the imaginary part of", and the bar means the "complex conjugate of".

The mechanical transients are obtained by combining Eq. 24 with

$$T=\frac{J}{q}\frac{d\omega_m}{dt}+\frac{D}{q}\omega_m+T_L \tag{25}$$

where T_L is the load torque. The currents and torque in Eqs. 22 through 24 are themselves functions of both ω_m and t. Though the electrical time constant is so small in comparison with the mechanical time constant, the rotor speed changes with time t for the transient. However, it is possible to numerically calculate motor transients by solving Eq. 22～25 simultaneously.

3. Measurements and Analytical Results

3.1 Measurements

All measurements and tests were performed on a 200V, 3-phase, 1.5kW, 6.6A, 50Hz, 4-pole squirrel cage induction motor. Conventional light running, locked rotor, and resistance tests were conducted in order to evaluate the electrical parameter of the machine, and a series of retardation and no-load mechanical loss tests were performed to evaluate the rotational moment of inertia and damping coefficient of the system. The electrical constants of the tested motor were $R_s=1.277\Omega$, $R_r=1.032\Omega$, $l_s=l_r=0.0041$H, and $L_m=0.0999$H. The mechanical constants of the motor were $J=0.0070$Nmsec²/rad²,

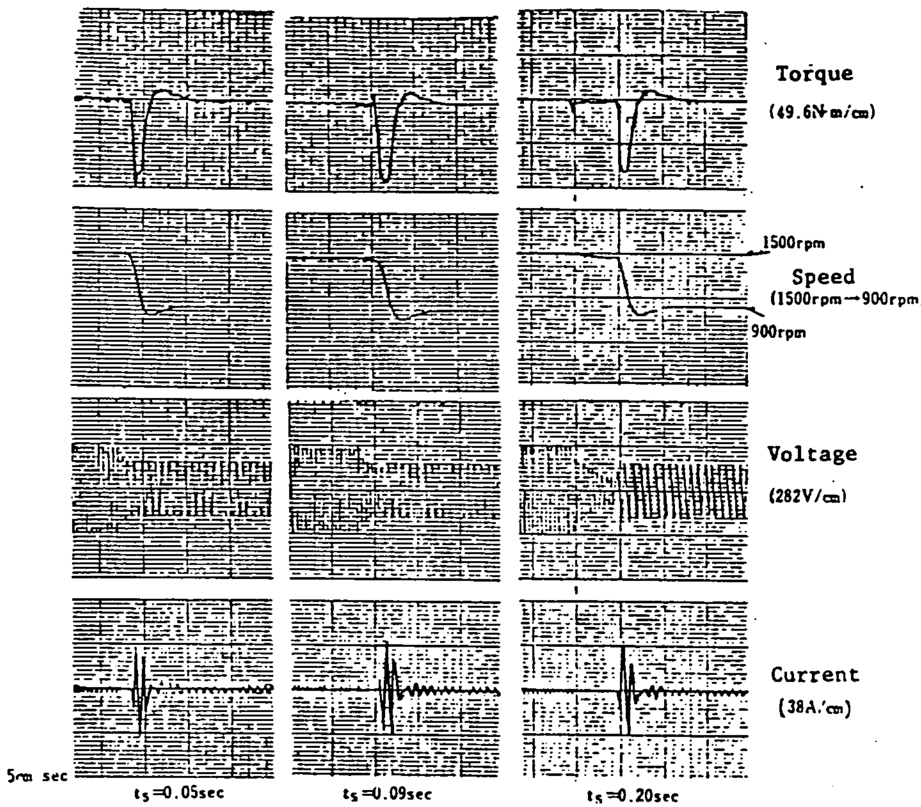

Fig. 2 Examples of measurement (200v50H→120v/30v/Hz)

$D=0.00073$Nmsec/rad and the induction motor coupled d. c. machine system had $J=0.084$Nmsec^2/rad^2 and $D=0.0061$Nmsec/rad. We used a 3-phase synchronous generator as a variable voltage variable frequency supply. The generator was rated at 10 KVA and its frequency could be varied from 10 to 100Hz. The transient torque was measured by attaching a magnetostrictive transducer to the shaft and sensing the voltage it induced in a stationary coil. The disconnect-reconnect operation was performed by manually operating a solid 3-phase knife switch. Obviously, the switch-off period could not be controlled, but because the reference phase voltage was recorded, the phase and the duration of switch-off could be determined. Experimental recordings of torque, speed, voltages, and currents of the induction motor coupled d. c. machine are shown in Figs. 2 and 3. From the results, the actual switching times could be accurately determined. These same values were used for the theoretical calculations in order to make the theoretical work consistent with the experimental results. After this, all measured and calculated results pertain to the case where the induction motor is coupled to a d. c. machine as an inertia load.

The measured speed-transient torque characteristics are shown in Figs. 4 and 5. The damping time constant of the residual voltage shown in Figs. 2 and 3 is equal to 0. 107

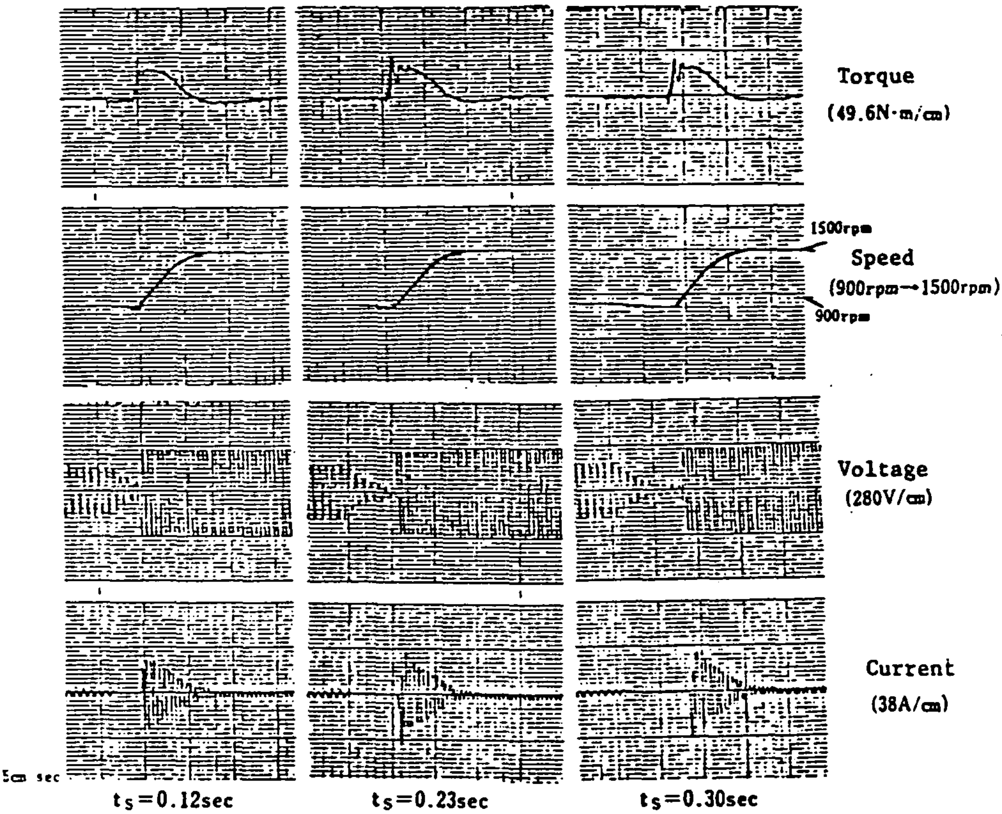

Fig. 3 Examples of measurement (120v/30Hz→200v/50Hz)

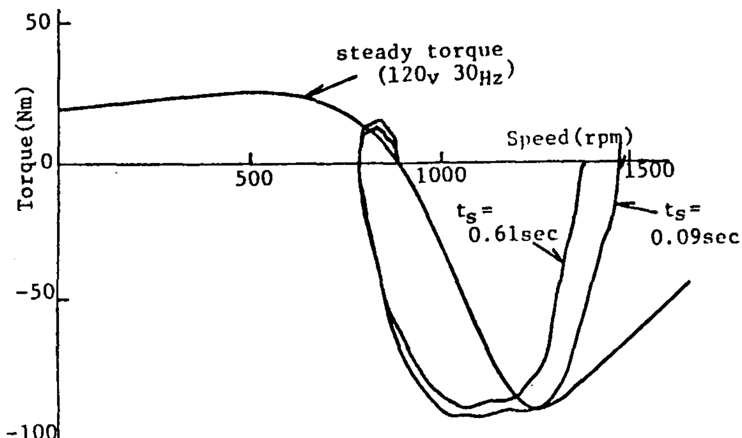

Fig. 4 Measured speed-transient torque characteristics
(200v/50Hz→120v/50Hz)

sec. The value calculated by using the electrical constants is 0.102 sec. Therefore, the measured value agrees with the calculated value.

3.2 Comparisons between measured and calculated results.

Figure 6 represents the transient torque of the induction motor due to reconnection

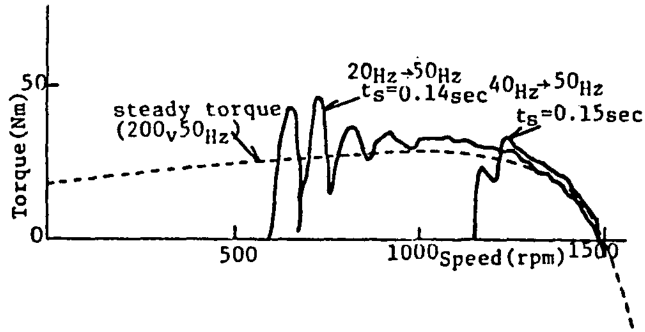

Fig. 5 Measured speed-transient torque characteristics (80v/20Hz, 160v/40Hz→200v/50Hz)

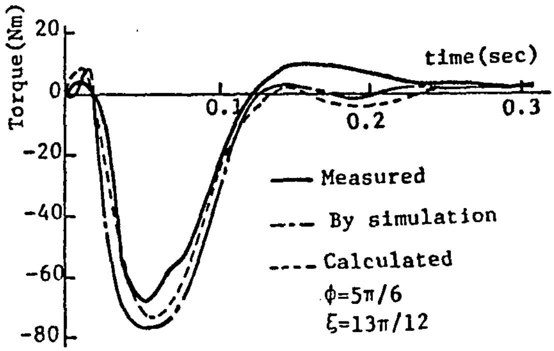

Fig. 6 Comparison between measured and calculated reconnection transient torque (t_s=0.042sec 200v/50Hz→120v/30Hz)

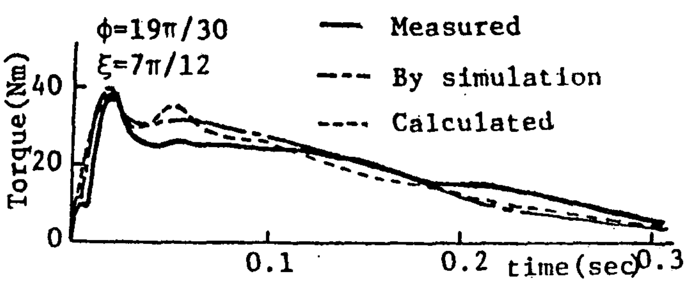

Fig. 7 Comparison between measured and calculated reconnection transient torque (t_s=0.044sec 200v/50Hz→280v/70Hz)

to a 120V, 30Hz supply disconnection from a 200V, 50Hz line. The switch-off interval was 0.042 sec. Figure 7 represents the transient torque due to reconnection of the 280V, 70Hz supply after disconnection from the 200V, 50Hz line. The switch-off interval was 0.044 sec. Figures 6 and 7 show that the numerical solutions of transient torque agree with the measured results, and also the calculated ones by d-q-o model simulations.

4. Discussion of Results

The authors discuss the transient stator current and torque due to a disconnect-reconnect operation. There are two cases of particular importance : (1) when the recon-

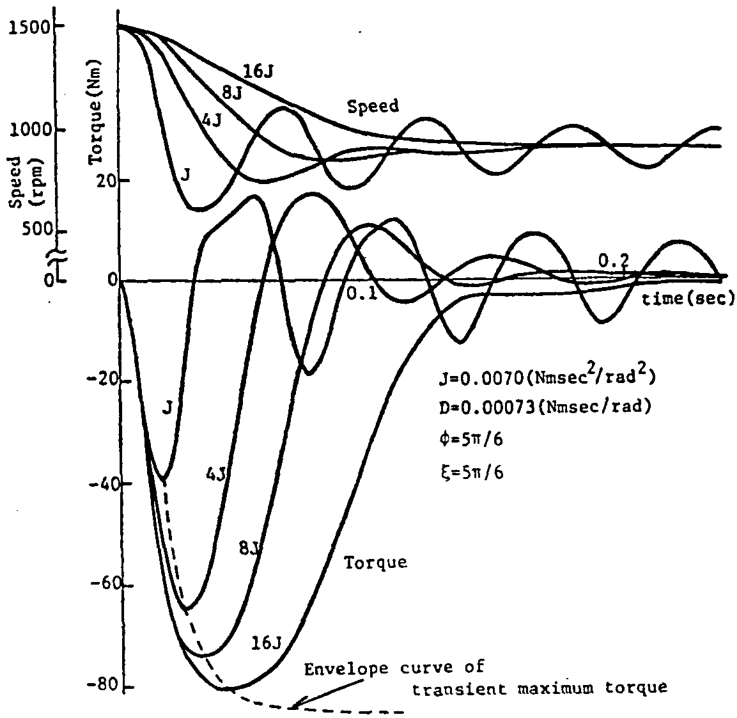

Fig. 8 Effects of moment of inertia on reconnection transient torques and speeds (t_s=0.043sec 200v/50Hz→120v/30Hz)

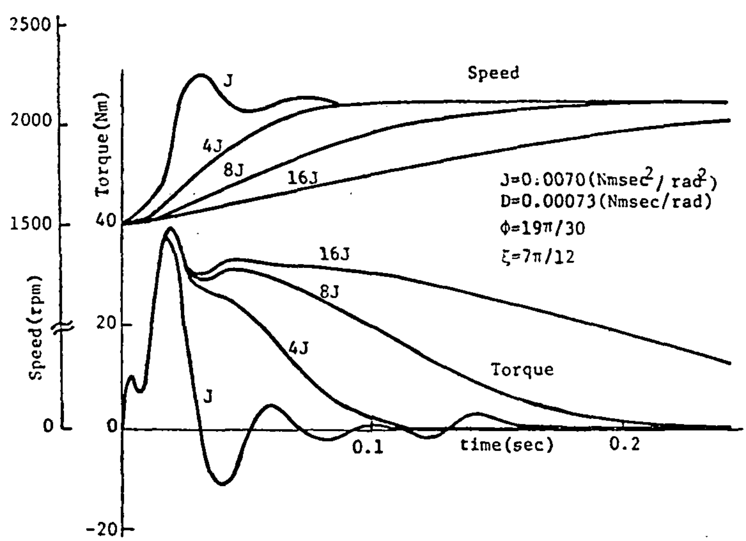

Fig. 9 Effects of moment of inertia on reconnection transient torques and speeds (t_s=0.044sec 200v/50Hz→280v/70Hz)

nected supply frequency, f_2, is smaller than the supply frequency, f_1, and (2) when f_2 is larger than f_1.

4.1 Transient torque wave forms and effects of Moment of Inertia.

In case of (1), it is found from Figs. 4 and 6 that a large negative torque occurs immediately after the instant of reconnection. This happens because the machine speed is above synchronous speed, and the motor acts as a generator. Figure 8 shows the calculated transient torques and speeds for different values of J with constant t_s, ϕ, and ξ following a disconnect-reconnect operation. Inspection of the transient torque in Fig. 8 shows that the moment of inertia influences the amplitude and the number of oscillations of the torque. In case of (2), one of the results is shown in Fig. 7. In this case, the machine operates as a motor in all the region from reconnection through steady operating state. Figure 9 shows the calculated transient torques and speeds for different values of J with constant t_s, ϕ, and ξ following a disconnect-reconnect operation. It is found from Fig. 9 that the positive transient maximum torque is independent of the moment of inertia.

4.2 Effects of the phase angle of the applied voltage at the instant of reconnection and the frequency.

Figures 10 and 11 show the calculated currents for different values of ξ with constant t_s, ϕ, and J following a disconnect-reconnect operation. It is found from these

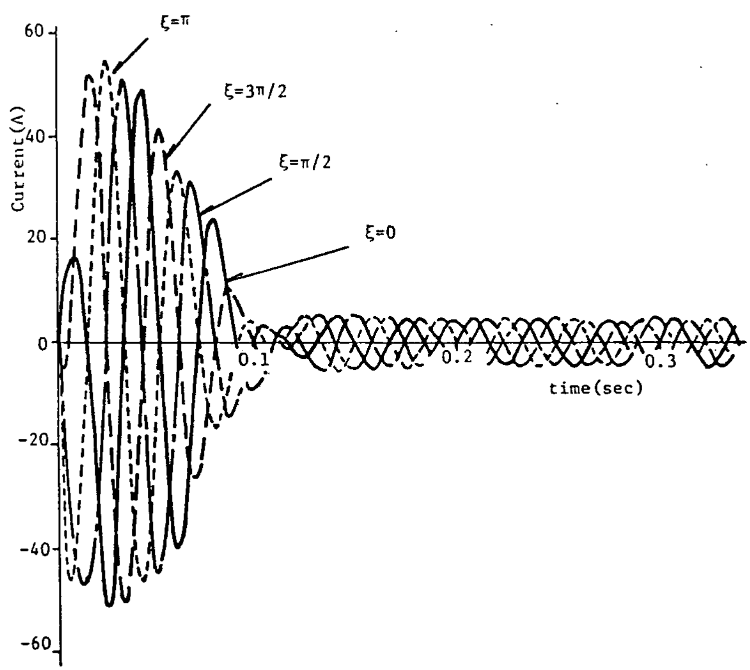

Fig. 10 Effects of phase angle ξ at the instant of reconnection on transient currents (t_s=0.1sec ϕ=0 200v/50Hz→120v/30Hz)

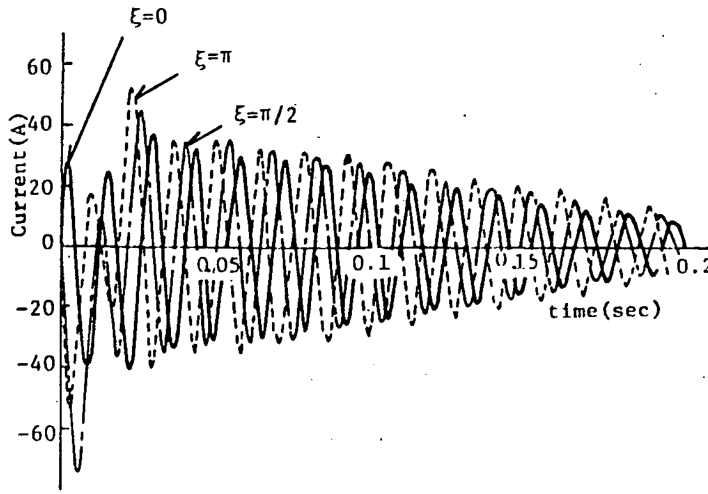

Fig. 11 Effects of ξ on reconnection transient current ($\phi=0$ $t_s=0.1$sec 200v/50Hz→280v/70Hz)

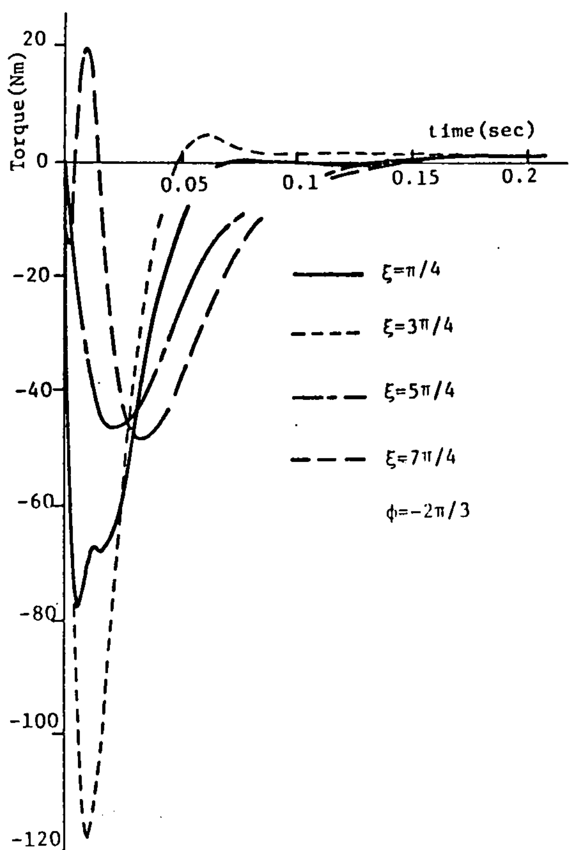

Fig. 12 Effects of ξ on reconnection transient torque ($t_s=0.013$sec 200v/50Hz→160v/40Hz)

two figures that the amplitude of the transient current immediately after reconnection is influenced by the phase angle of the applied voltage at the instant of reconnection.

Figures 12 and 13 show the calculated transient torque for different values of ξ with

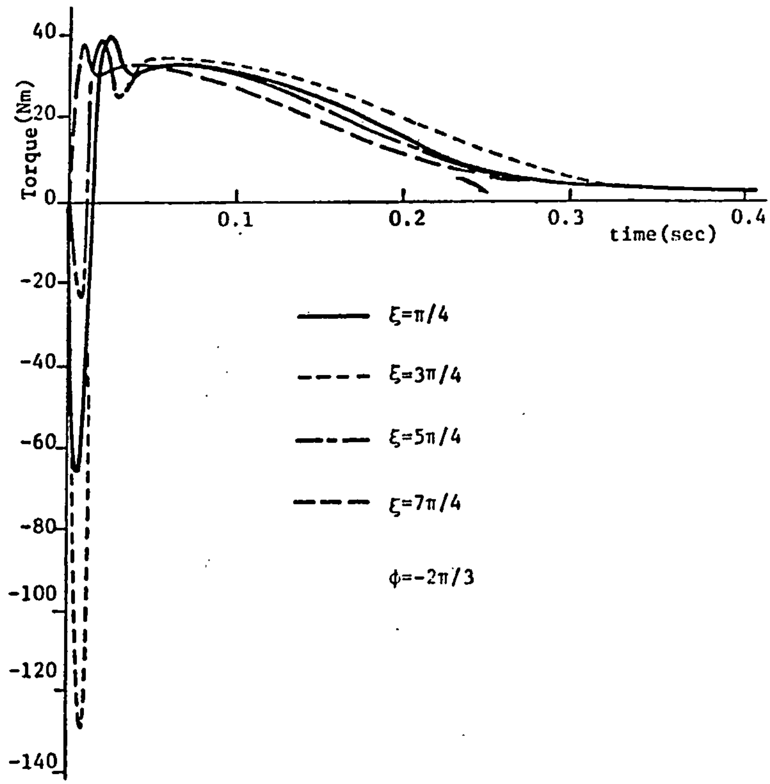

Fig. 13 Effects of ξ on reconnection transient torque (t_s=0.012sec 200v/50Hz→280v/70Hz)

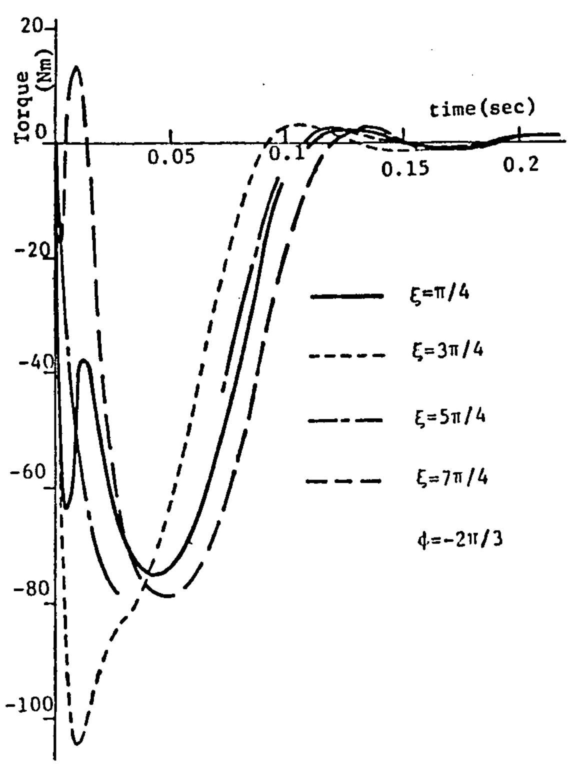

Fig. 14 Reconnection transient torque (t_s=0.013sec 200v/50Hz→120v/30Hz)

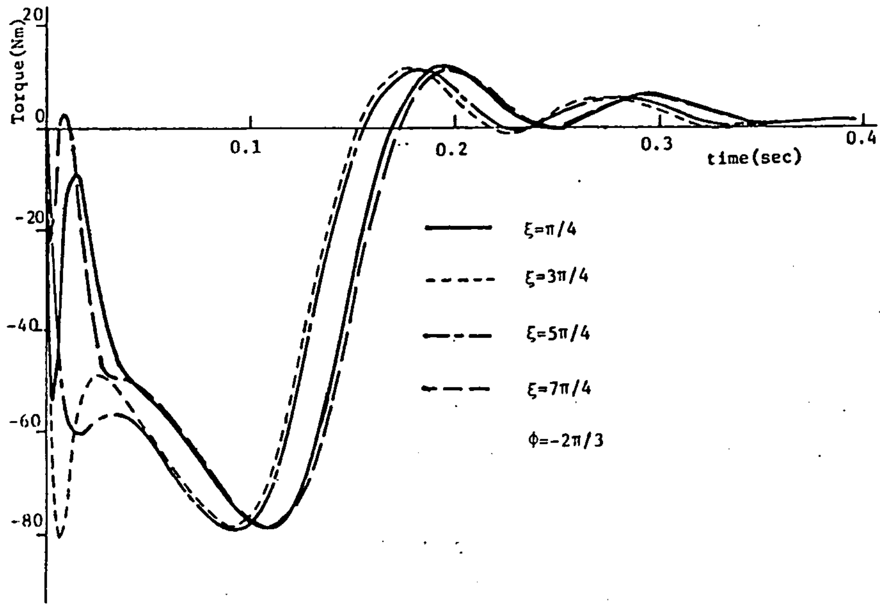

Fig. 15 Reconnection transient torque (t_s=0.013sec 200v/50Hz→80v/20Hz)

constant t_s, , ϕ, and J following a disconnect-reconnect operation. It is apparent from Fig. 12 that the transient negative peak torque, T_p, is influenced considerably by ξ, and the maximum value of T_p eventually developes to about triple the minimum value of T_p with ξ. It is found from Fig. 13 that the negative transient torque is influenced considerably by ξ, but the positive transient torque is independent of ξ. As shown above, the maximum transient torque eventually developes to nearly ten times the rated torque.

For different values of f_1, f_2, and its ratio, we had a lot of measured and calculated data. Figures 14 and 15 respectively are representative. As a result of considering many data, it was found that, if f_1/f_2 were nearly equal to 0.8, the transient negative peak torque depends strongly upon ϕ and ξ. In contrast, when f_2/f_1 was around 0.4, T_p was independent of ϕ and ξ. It became clear that the duration of the transient state was independent of the phase angle of the applied voltage, and the phase angle and the amplitude of the residual voltage, but it was a function of f_1/f_2 only.

5. Conclusions

In this study, we have investigated the influence of the applied voltage on the transient behavior of an induction motor following a disconnect-reconnect sequence. In order to analyze the resulting transients, the positive-sequence differential equations for the stator and rotor currents and the equation for the instantaneous torque were represented by using the instantaneous symmetric coordinate method. The transient currents and torques for the variable rotor speed were obtained by solving the differential equations under appropriate initial conditions and evaluating together with the motional equation by digital computer using Runge-Kutta method. In particular, the effects of the phase angles and frequency of the reapplied voltage were studied in great detail. The hibrid solution of this problem obtained by this method agreed with the measured and the calculated results by d-q-o model simulation.

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Used Symbols

All symbols are in S. I. units.

D: Damping coefficient

e_a: Instantaneous residual voltage ot reference phase.

e_{v1}: Positive–sequence component of the motional e. m. f.

E: R. M. S. stator phase voltage before disconnection from the supply.

i_s: Instantanəous current in the primary.

i_r: Instantaneous current in the secondary refered to the primary,

i_{s1}: Instantaneous value of the positive-sequence component of i_s.

i_{r1}: Instantaneous value of the positive-sequence component of i_r.

i_{ra}: Instantaneous current in the rotor while the residual voltage is produced.

i_{s10}: Initial value of i_{s1}.

i_{r10}: Initial value of i_{r1}.

I_{s1}: Laplace transform of i_{s1}.

I_{r1}: Laplace transform of i_{r1}.

J: Moment of inertia.

l_s: Primary leakage inductance per phase.

l_r: Secondary leakage inductance per phase.

L_m: Maximum mutual inductance per phase between primary and secondary windings.

$L_s=l_s+L_m$: Primary self-inductance per phase.

$L_r=l+_rL_m$: Secondary self-inductance per phase.

$p=d/dt$: Differentiation operator.

q: Number of pole pairs.

R_s: Primary resistance per phase.

R_r: Secondary resistance per phase.

s: Complex-variable.

S: Slip

T: Developed instantaneous torque.

t: Time referred to the instant of reconnection.

t': Time referred to the instant of disconnection.

t_s: Time taken for the changeover from one supply to the other.

v_{s1}: Instantaneous value of the positive-sequence component of applied stator phase voltage.

V_1: Reconnected R. M. S. stator phase voltage.

ξ: Initial phase angle of the reconnected voltage.

ϕ: Initial phase angle of residual voltage referred to the instant of reconnections.

ϕ': Initial phase angle of residual voltage referred to the instant of disconnections.

ω: Angular frequency of the reconnected voltage.

ω': Angular frequency of applied voltage before disconnection.

ω_m: Electrical angular velocity of the rotor.